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GEOMETRIC DESCRIPTION AND GENERATION OF SURFACES.(U)
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by

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Abstract

A geometric technique for the description and generation of arbitrary doubly-curved surfaces is given. The technique is based on a curve generating algorithm derived from Bezier and the author, which is described. A surface is then determined to be the set of surface curves defined by a set of generator curves. Finally, some features of surfaces of this type are examined.

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Geometric Description and Generation of Surfaces

This report describes recent work at New York University on the description and generation of arbitrary doubly-curved surfaces by solely geometric means. We believe this approach offers a variety of advantages over previously developed techniques for computer-aided design which are based on algebraic analysis of the functions which describe the surface.

A geometric approach has the primary advantage that its techniques (compass and straight-edge constructions) are familiar to all the design professions, while the algebraic methods employed in previous systems frequently are not familiar to broad sections of these professions. We believe that a tool whose mechanism is understood will be used with greater ease, flexibility, and creativity than a "black box".

A second advantage to a geometric approach is that it appears to have as many degrees of freedom as the most general surface developed to date: the Coons surface. As a construction, the surface is independent of choice of axis and may be multivalued, closed or self-intersecting.

If the construction is considered as taking place in a lattice, then the construction is simultaneously the complete set of instructions required to drive numerous incremental devices, notably plotters and raster displays, numerical control machines, and various industrial robots.

The surface description is based on a curve construction technique similar to one described by the author in (2), but which has been extended to N-sided polygons as explained below. A surface will then be defined to be the family of <u>surface curves</u> whose construction is determined by a set of <u>generator curves</u> constructed by the same method.

Curve Description

A curve is described by an N-sided polygon (open or closed) whose sides are ordered 1 to N. Join the midpoint of side 1 to the midpoint of side 2, the midpoint of side 2 to that of side 3, ... , the midpoint of side N-1 to that of side N, forming an N-1 sided polygon. Repeat this procedure on the resulting N-1 sided polygon, yielding an N-2 sided polygon. N repetitions of this procedure determines a point on the curve, and this point is the endpoint of two N-sided polygons (see Fig. 1). Since the curve described by the polygon contains the endpoints of the polygon, the above procedure is applied recursively to the generated polygons until a point within the desired limit is obtained. The curve is the ordered set of all such points.

Geometrically this construction is classed as an "affine construction"; a construction requiring only a straight edge, but where the straight edge is permitted to slide in order to generate parallels. This capability replaces the necessity for a compass to divide the line in half.

The curve described by an N-sided polygon is an N-space curve, single-valued in N-space, but possibly multivalued in spaces of dimension less than N.

This construction is derived from two previously developed constructions, described by the author (2) and Bezier (1). The method outlined in (2) is similar to the construction above, but limited to the case with two-sided polygons. In that case, a three-sided polygon is divided along the middle leg into two 2-sided polygons, each of which is treated as above (see Fig. 2). Bezier's construction applied to N-sided polygons as follows: If we wish to find a point on the curve corresponding to a particular parameter value (where the curve is considered a function of a parameter varying between 0 and 1) of ¼, we find a point on each leg ¼ of the distance to the next leg. Connecting these points, we construct the N-1 sided polygon. We repeat

this process on the resulting polygons until we have just one point. This is the point on the curve corresponding to the parameter value $\frac{1}{2}$.(see Fig. 3). Given a polygon describing a curve, Bezier's construction can be used to approximate the curve by iterative application of the construction for various sequential values of the parameter.

In this context, our construction can be described as follows: First we determine a point corresponding to a parameter value of $\frac{1}{2}$. Having done this, we have also defined two new N-sided polygons whose endpoints are on the curve. We find the point corresponding to $\frac{1}{2}$ for each, and so on, recursively.

Curve Computation

The computational interpretation of the geometric construction above proceeds as follows: The N-sided polygon is represented by the N+1 ordered sequence of pairs or triples representing the coordinates of the vertices. If we add each point to the next sequential point (i.e., we add their coordinates) and divide by 2 (a right shift), we generate the sequence (with one less point) of midpoints. Repeating this procedure N times, each time saving the first and last points of each sequence, we get both a first-point sequence and a last-point sequence, each of N+1 points. The points of these sequences describe the two N-sided polygons generated from the first polygon. The last point of the saved set of "first points" and the first point of the saved set of "last points" are coincident; this point is on the curve.

If the distance of this coincident point from the first of the "first points" is greater than the maximum tolerance we are seekeing between successive points on the curve, we stack the set of "last points" and apply the procedure described above recursively to the set of "first points" until a point within the limit is found. Finding such a point, we pop a set off the stack, and repeat the overall procedure. See Fig. 4.

If we regard this as taking place in an integer raster space, and if A is a point on the curve, we cannot find another point on the curve closer than 2^{N-1} units away from point A. This occurs as a result of the fact that between successive approximations, N divisions take place. Since each division truncates, coordinates within that range will be identical with those of point A. If all coordinates are within a 2^{N-1} range of A, the only approximation we can make is A itself. We can, however, approach a uniform limit regardless of the degree of the polygon if we multiply (scale) each point by 2^{N-1} (N-1 left shifts) at the outset, and then divide each pointon the curve by 2^{N-1} (N-1 right shifts) upon completion.

An important advantage of this technique is that it involves only adds, shifts, and compares, and, therefore, is both fast and efficient in execution.

Surface Patch Description

We shall now use the curve description and generation method outlined above to define and generate a surface. Specifically, we shall define a set of curves, the points of which will said to describe a family of curves on the surface. If the surface is defined in a three-dimensional raster space, then every point on the surface is intersected by at least one curve.

Bezier (1) describes this as the generalization of his UNISURF method, from which it differs in that the set of curves defining the family of surface curves in UNISURF must all be of the same degree (or equivalently, be described by polygons with the same number of sides).

Consider a surface described by an ordered set of curves $\{A_i, 0 \leq i \leq N\}$, each defined by our curve generator over the same raster space. Each such curve consists of an ordered sequence of points on the curve. Therefore, we may form an ordered sequence of N+1 tuples such that the ordering

of the N+1 tuples is determined by the ordering of the points constituting the set of curves $\{A_i\}$. Let the j-th N+1 tuple define a curve B_j. The surface can then be represented by the set of curves $\{B_j\}$. See Fig. 5. The boundary curves of this surface patch are the curves A_0, A_N, B_1, and B_M where M is the index of the last N+1 tuple.

Since in a finite raster space A_k might have more (or fewer) points than A_1, a complete and "well-ordered" set of N+1 tuples is obtained by computing the curves $\{A_i\}$ in parallel and "lock-step" (i.e., applying the convergence criterion to the set of curves rather than just one).

This method is powerful in its surface-generating capability because the curves $\{A_i\}$ (called "generator curves") can be of arbitrary degree, whereas the curves $\{B_j\}$ (called "surface curves") are all of degree N. See Fig. 6.

Surface Patch Generation

At New York University, we have implemented the surface patch description on an Adage AGT-30 graphics computer in a prototype system which allows us to describe and generate a patch as a sequence of three-dimensional vectors constituting the surface curves of the patch. The AGT-30, with a hardware display processor which transforms an image file of three-dimensional vectors into a two-dimensional CRT image of the object from any viewpoint, permits us to rotate the surface with a joystick and to display different aspects of it.

The procedure used is simple, involving basically just two levels of the curve-generating algorithm, and is implemented in FORTRAN. The higher-level routine functions as a many-curve generator, and computes an arbitrary number of curves of various degrees as the "generator curves" in parallel. If one curve converges more rapidly than the others, computation on it is suspended. When all curves have converged, the N+1 tuple (consisting of

the points found on the N+1 generator curves) is passed to the lower-level curve generator, which computes the sequence of three-dimensional vectors constituting a surface curve B_j of degree N.

Smoothing Across Patch Boundaries

Adjacent patches share a common boundary curve, but this alone does not insure that the set of curves lying on the surface of each patch will meet smoothly across this common curve. Although it will be possible for a designer to define adjacent patches in such a way that they will be smoothly connected, the additional steps the designer would take to accomplish this can be automated as follows: Take the two (generator or surface) curves on adjacent patches which share a common point on the boundary. Select the point on each curve which is sequentially adjacent to this common point. Find the midpoint between them, and reflect it about the common point. Next find the midpoints between this new point and each of the points sequentially adjacent to the common point. The points thus generated are added to the definition of each curve such that they become the sequentially adjacent points to the common point. This will result in first and second derivative continuity across the common boundary. See Fig. 7.

It should be noted that the above approach will result in small changes in the surface shape from what was originally defined for each patch. This is the adjustment required to fit the two patches together smoothly. See Fig. 8.

Riesenfeld (3) has shown that low-order curves (e.g., cubic curves) conform much more closely to their defining polygons than high-order ones, and points out that this is of considerable advantage to a designer. The smoothing procedure above suggests that a designer could "sketch" a surface as local low-order patches which then could be "smoothed" together as needed.

Features of a Surface

Another advantage of the uniform distribution of the points on a surface patch is the ease with which it allows us to determine important engineering characteristics of the surface, such as its area, bounded volume, and moment of inertia. Each point is either 1, $\sqrt{2}$, or $\sqrt{3}$ units distant from each adjacent point. This means that the surface may be regarded as a polyhedron whose facets are chosen from a small set of right triangles of known dimensions.

For purposes of display, this capability to resolve to the limit of an integer raster space can be mapped directly into the resolution limit of the display device. This means that an object displayed can be portrayed simply, but as smoothly as the device is capable of in terms of shape, intensity, color, etc. Similarly, in a context of numerical control devices, it can readily define a surface as smoothly as the device can machine it.

Conclusion

Simple geometric procedures are sufficient for the description and generation of complex doubly-curved surfaces. The significance of this is that: 1. The method used can be readily and completely understood by the wide variety of designers who might use it. 2. Its' simplicity is directly translateable into simple and efficient machine code, including implementation in hardware or microprocessor format. 3. The affine construction employed in the algorithms provides an interesting, and possibly fruitful, mathematical perspective on the other mathematical techniques used to generate surfaces. 4. The surface is described in a regular and predictable format, from which additional information or benefits may be derived.

References

(1) Bezier, P. Numerical Control - Mathematics and Applications (trans. Forrest, Pankhurst) London: John Wiley & Sons, 1972

(2) Chaikin, G. "An Algorithm for High-Speed Curve Generation" Computer Graphics and Image Processing, 3, (4), December 1974

(3) Riesenfeld, R. "Applications of B-Spline Approximation to Geometric Problems of Computer-Aided Design", UTEC-CSc-73-126, Computer Science, University of Utah, March 1973

Bibliography on Computer-Generated Curves and Surfaces

1. S. A. Coons, "Surfaces for Computer-Aided Design of Space Forms," M.I.T. Project MAC, MAC-TR-41, June 1967.

2. A. R. Forrest, "Curves and Surfaces for Computer-Aided Design," Ph.D. dissertation, University of Cambridge, England, July 1968.

3. A. R. Forrest, "Interactive Interpolation and Approximation by Bezier Polynomials," Computer Journal, 15, 1972, 21-79.

4. W. J. Gordon and R. F. Riesenfeld, "Bernstein-Bezier Methods for the Computer-Aided Design of Free-Form Curves and Surfaces," J.ACM, 21, (2), 1974, 293-310.

5. R. F. Riesenfeld, "Applications of B-Spline Approximation to Geometric Problems of Computer-Aided Design," UTEC-CSc-73-126, Computer Science, University of Utah, March 1973.

6. P. Bezier, Numerical Control - Mathematics and Applications, (trans. Forrest, Pankhurst) London: John Wiley & Sons, 1972.

7. G. Chaikin, "An Algorithm for High-Speed Curve Generation," Computer Graphics and Image Processing, 3, (4), December 1974.

8. D. B. Almond, "Numerical Control for Machining Complex Surfaces," IBM Systems Journal, 11, (2), 1972, 150-168.

9. D. V. Ahuja, "An Algorithm for Generating Spline-Like Curves," IBM Systems Journal, 7, (3 and 4), 1968, 206-217.

10. S. A. Coons and B. Herzog, "Surfaces for Computer-Aided Design," AIAA paper no. 67-895, Am. Inst. Aeronautics and Astronautics, New York, Oct. 1967.

11. T. M. Lee, "Three-Dimensional Curves and Surfaces for Rapid Computer Display," Ph.D. dissertation, Div. Eng'g. and Appl. Phys., Harvard University, March 1969.

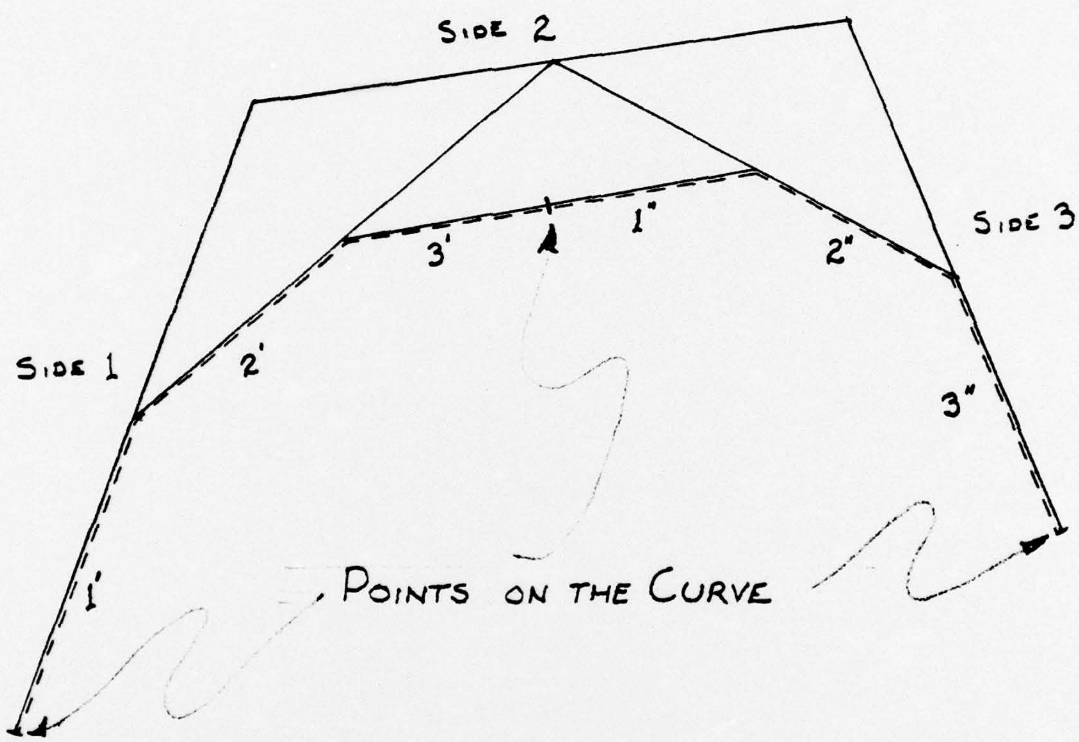
12. A. R. Forrest, "Curves for Computer Graphics," in Pertinent Concepts in Computer Graphics, ed. by M. Faiman and J. Nievergelt, University of Illinois Press, Urbana, Ill., 1969, 31-47.

13. A. R. Forrest, "On Coons and Other Methods for the Representation of Curved Surfaces," Computer Graphics and Image Processing, 1, (4), December 1972, 341-359.

14. G. J. Agin, "Representation and Description of Curved Objects," Ph.D. dissertation, Comp. Science Dept., Stanford Univ., Oct. 1972 (AD 755 139)

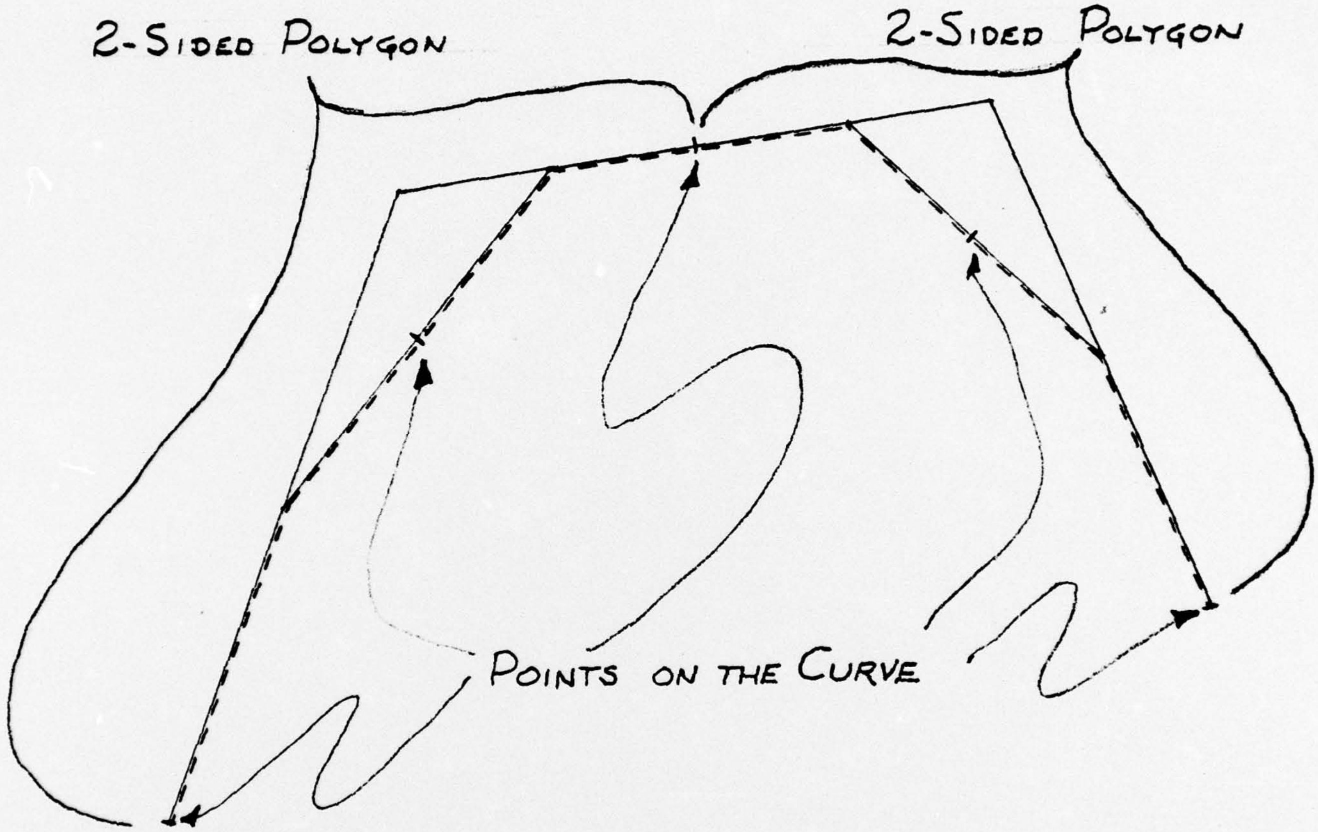
15. D. Cohen and T. M. Lee, "Fast Drawing of Curves for Computer Display," Proc. Spring Joint Comp. Conf. 1969, AFIPS, 34, 1969, 297-307.

16. A. A. Ball, "CONSURF Part one: Introduction to the Conic Lofting Tile," Computer-Aided Design, 6, (4), Oct. 1974, 243-249.



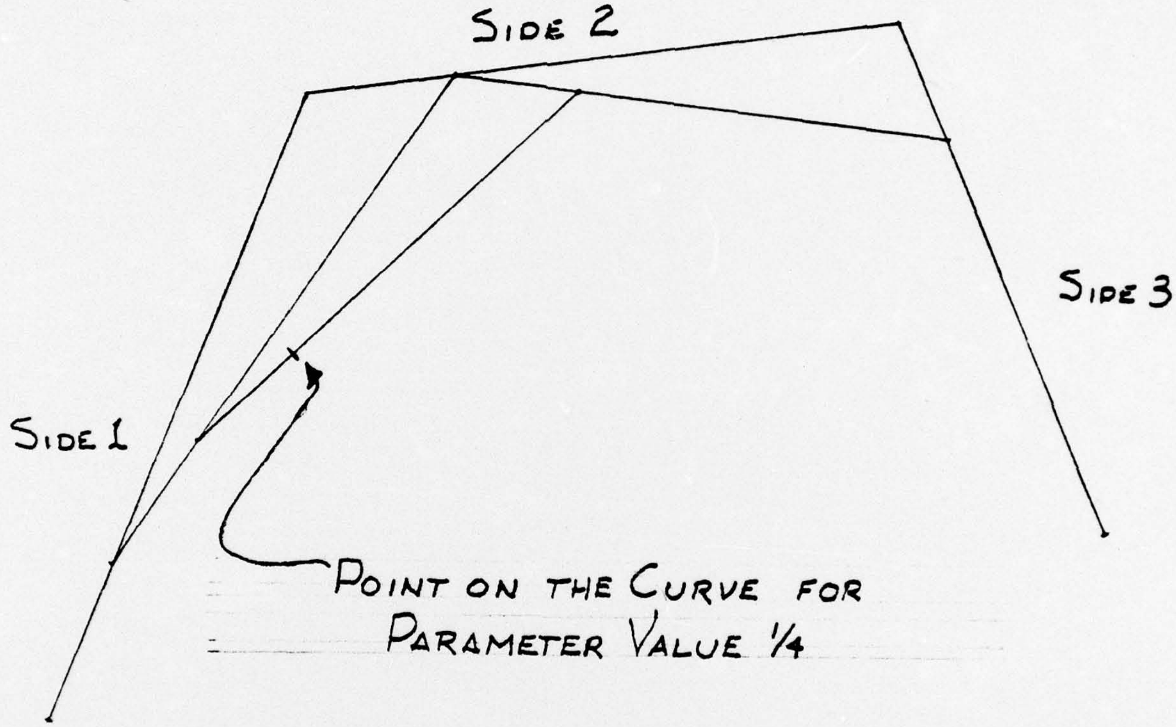
Construction determines a point on the curve and defines two new polygons

Figure 1



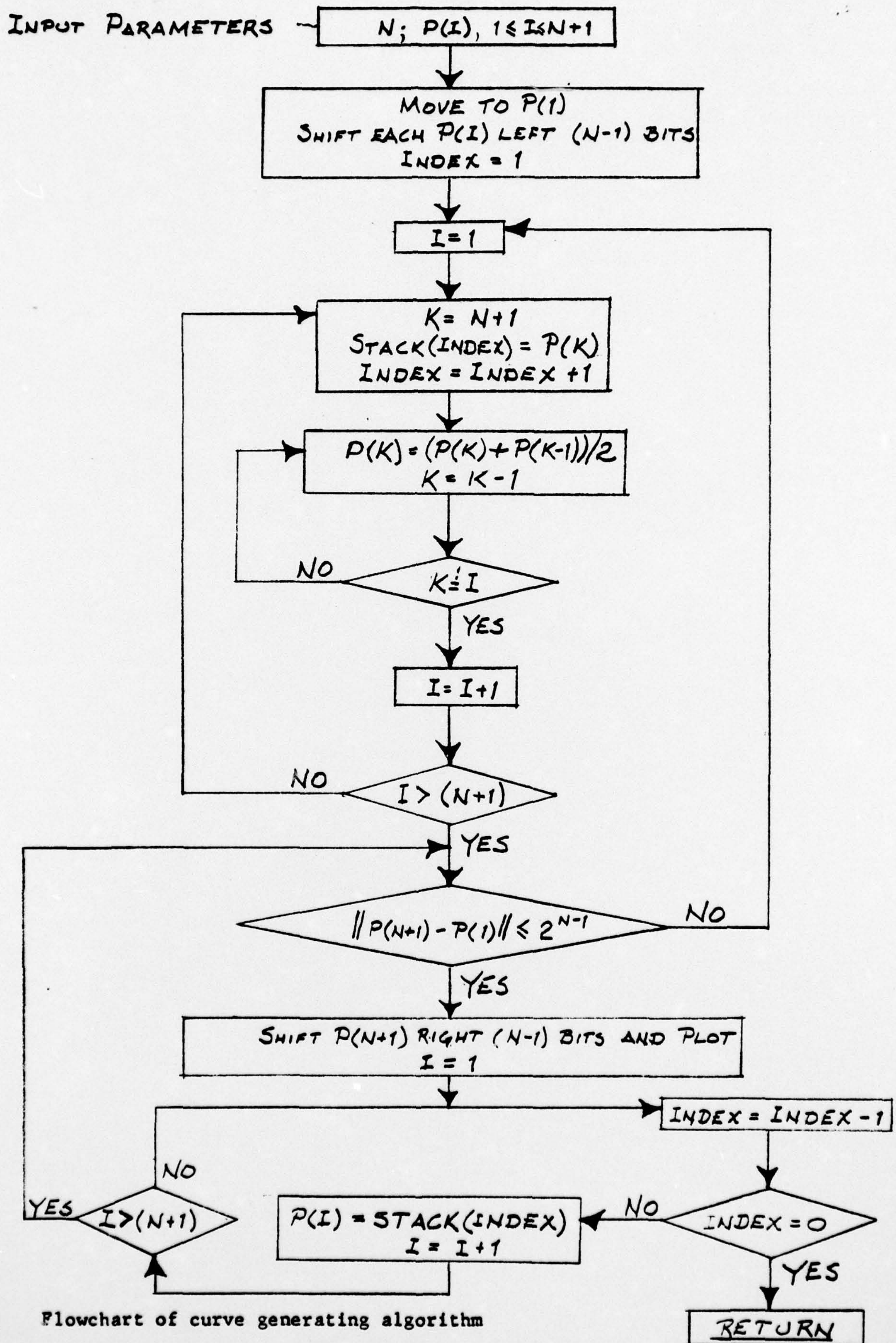
Construction which results in two piecewise parabolic curve segments

Figure 2



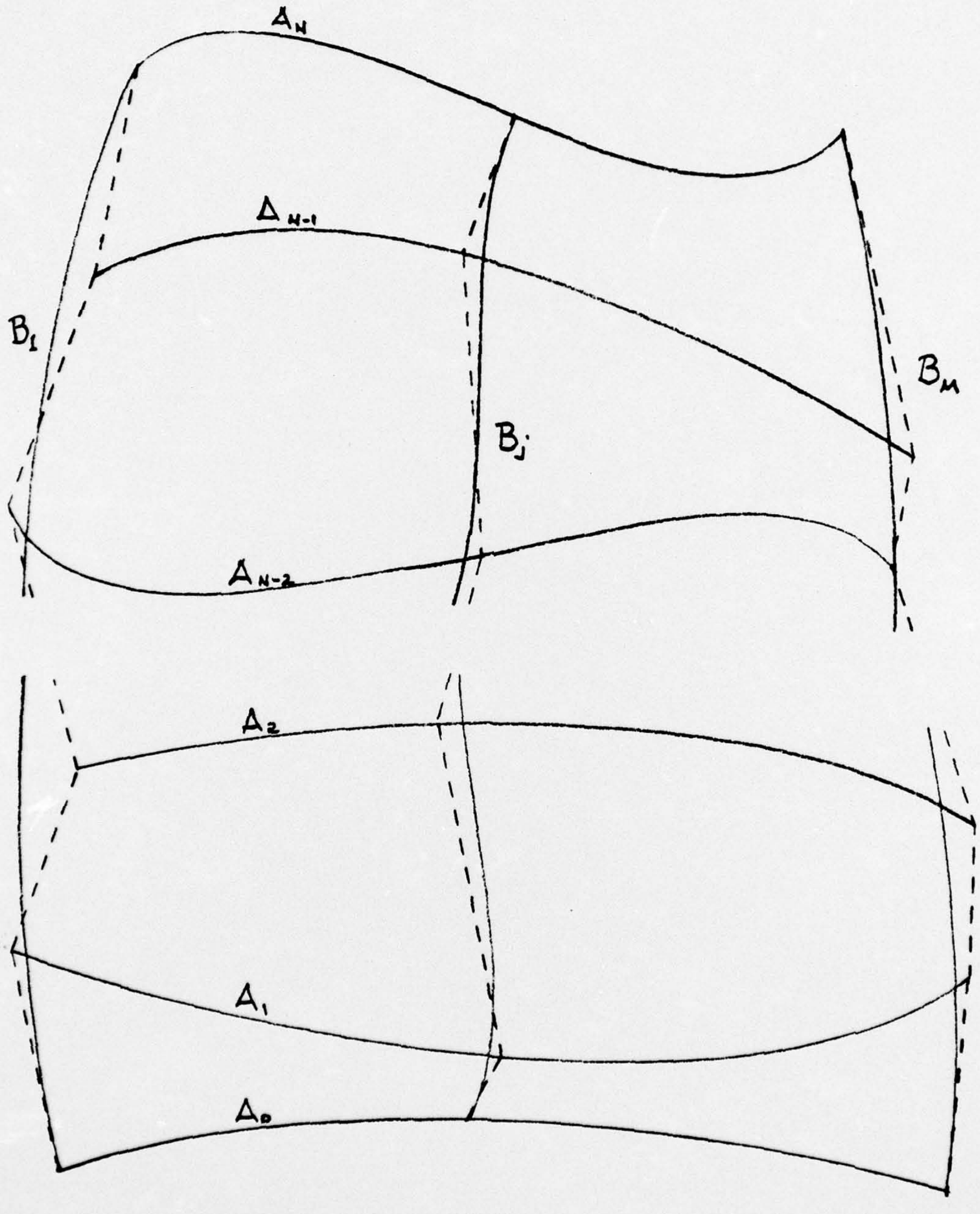
Bezier's geometric construction

Figure 3



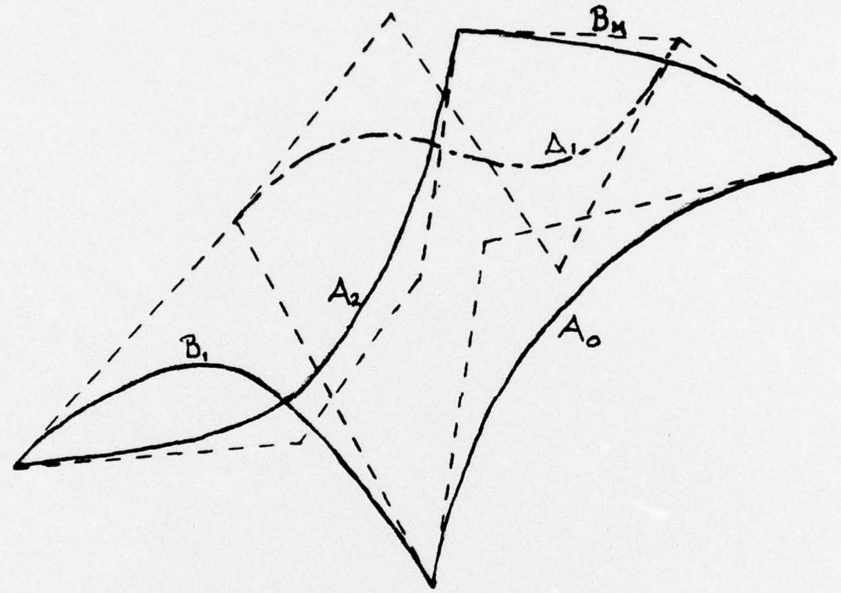
Flowchart of curve generating algorithm

Figure 4



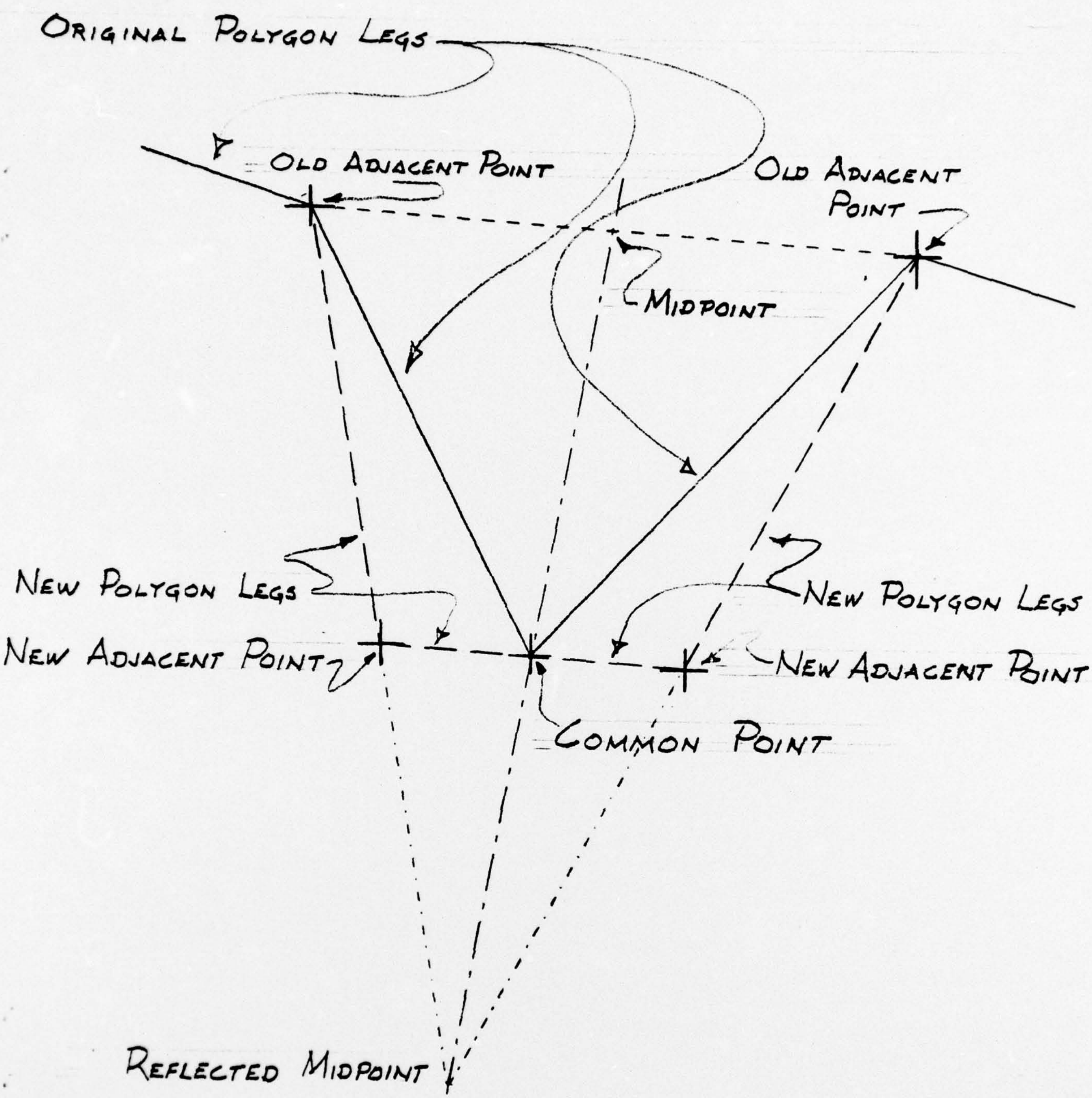
Surface with generator curves A_i and surface curves B_j

Figure 5



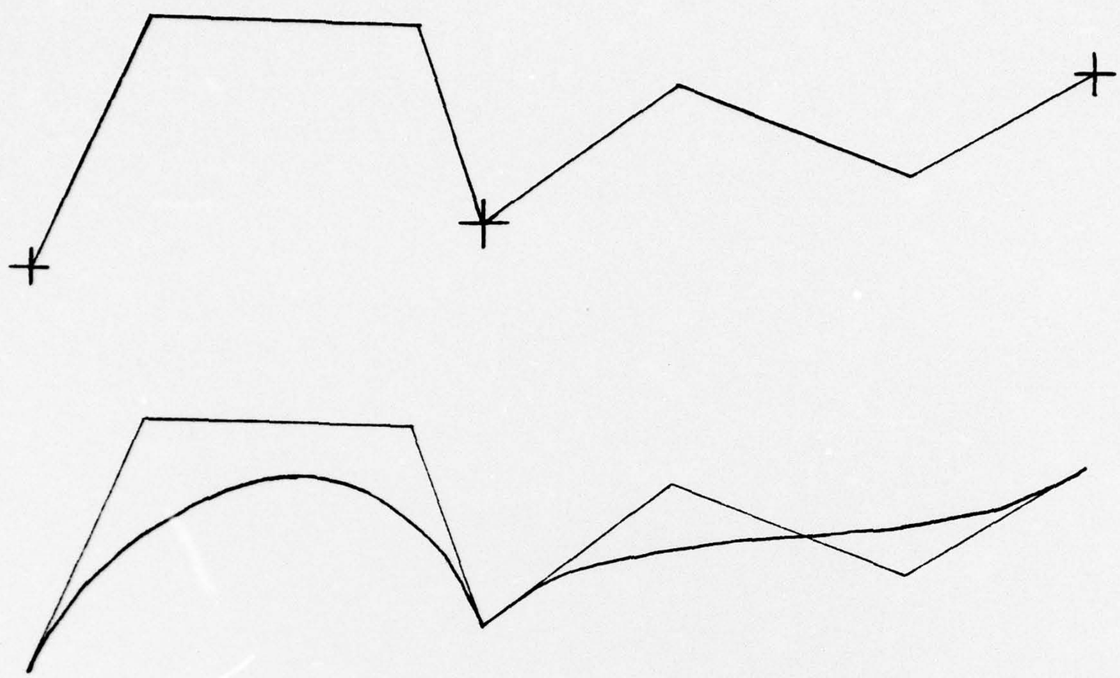
Parabolic surface curves defined by generator curves of various degrees

Figure 6

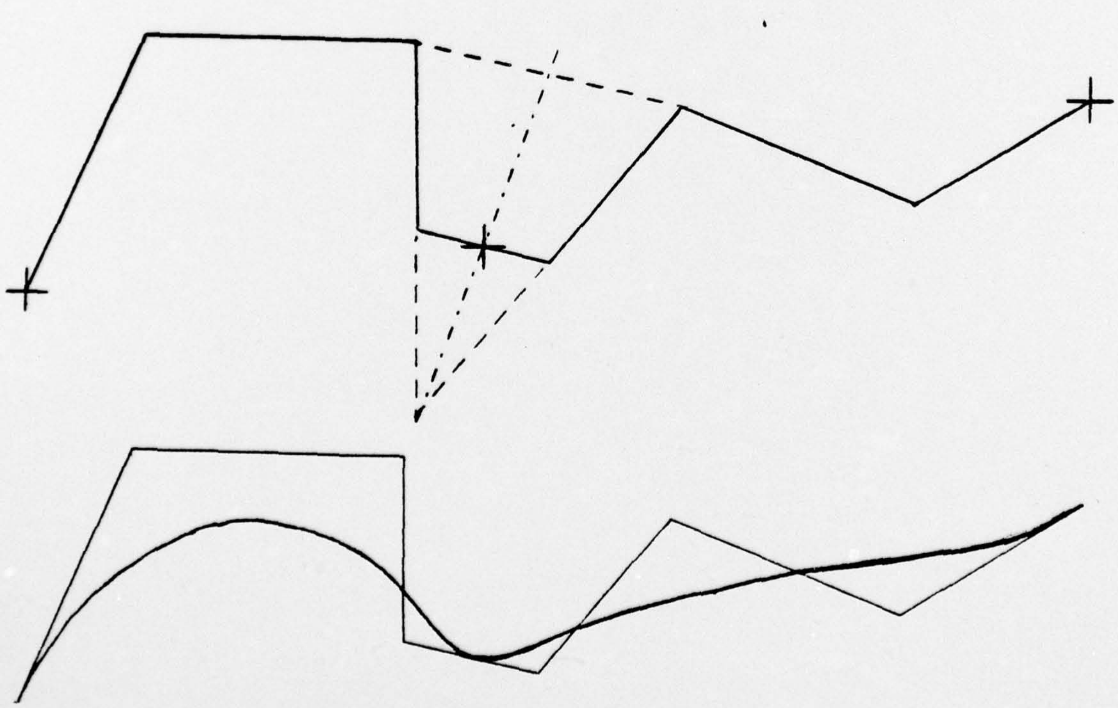


Construction which blends two adjacent curves with second derivative continuity

Figure 7

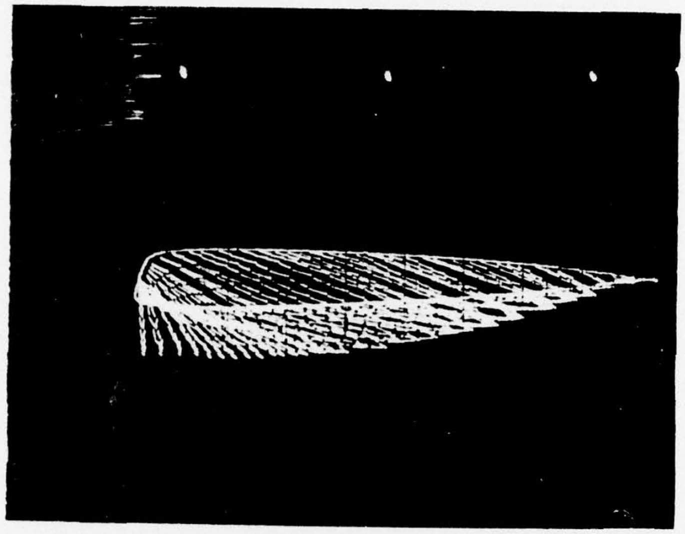


a) Two adjacent curves



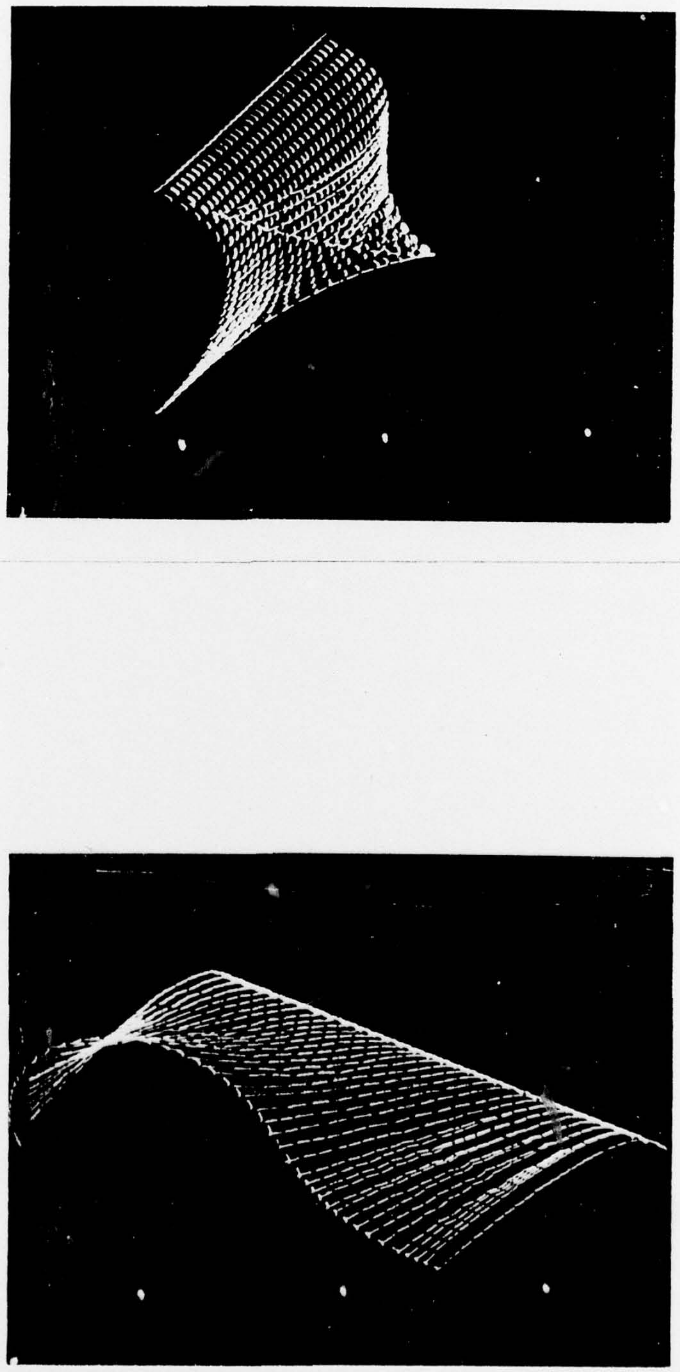
b) Two curves after blending

Figure 8



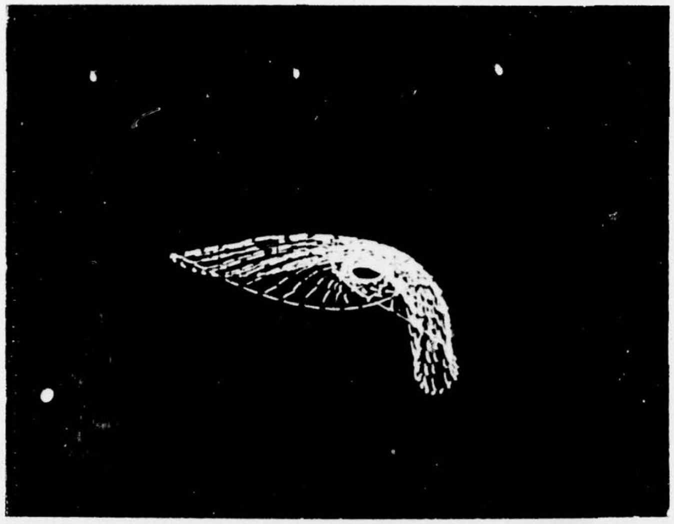
Boat hull defined by a single surface patch

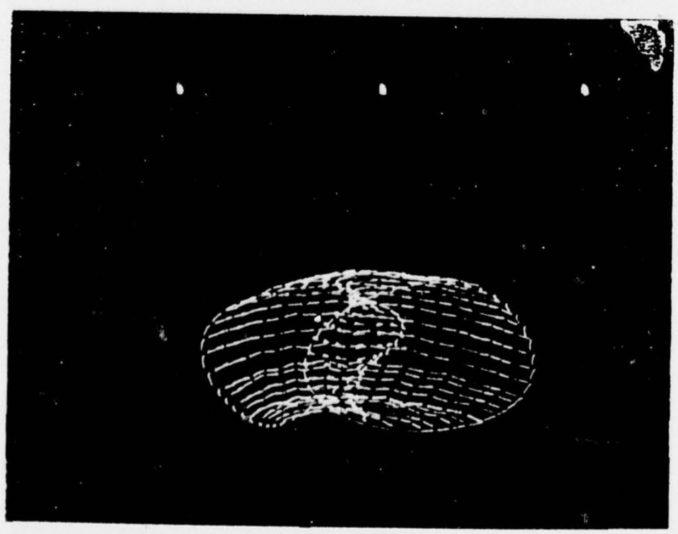
Figure 9



Two views of an automobile fender defined by a single surface patch

Figure 10





Two views of a self-intersecting surface defined by a single surface patch

Figure 11

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Curve fitting

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